Mechanica 7a (7P500)

Stability

Uitwerking opdrachten

Technische Universiteit Eindhoven
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Studiejaar 2005-2006

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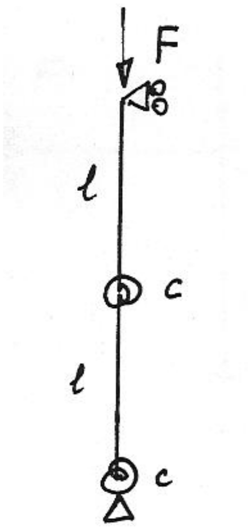
Opdracht 1

Opdrachtformulering

- Bepaal met behulp van de evenwichtsmethode en potentiële energie de kniklast van onderstaande schema. Zet de resultaten uit in een matrix geometrisch en materiaal / lineair en niet-lineair.



Uitwerking

Kniklast met behulp van potentiële energie

Inwendige potentiële energie

$$E_{pot;i} = \frac{1}{2}c(2\phi)^2 + \frac{1}{2}c(\phi)^2$$

Uitwendige potentiële energie

$$E_{pot;u} = -Fu = -F \cdot 2l \cdot (1 - \cos\phi)$$

Totale potentiële energie

$$E_{pot} = E_{pot;i} + E_{pot;u} = 2\tfrac{1}{2} \cdot c(\phi)^2 - F \cdot 2l \cdot (1 - \cos\phi)$$

$$E_{pot} = 2\tfrac{1}{2} \cdot c(\phi)^2 - F \cdot l \cdot (\phi^2)$$

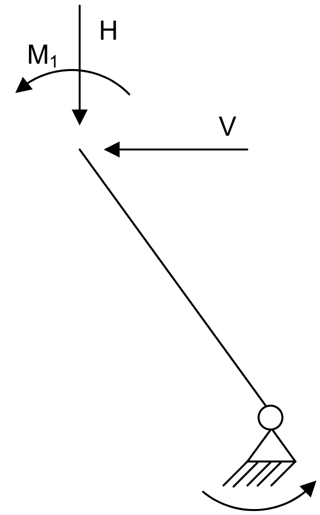
$$\delta E_{pot} = 0 \quad \rightarrow \quad \frac{\delta E_{pot}}{\delta\phi} = 0$$

$$\delta E_{pot} = 5c\phi - 2Fl\phi = 0 \quad \rightarrow \quad \phi \neq 0$$

$$\delta E_{pot} = (5c - 2Fl)\phi = 0$$

$$(5c - 2Fl)\phi = 0 \quad \rightarrow \quad \boxed{F = \frac{5}{2} \cdot \frac{c}{l}}$$

Kniklast met behulp van evenwichtsmethode



$M_2 = F \cdot w + M_1 - V \cdot l$

Equilibrium: $F \cdot w = M_1 + V \cdot l$

Kinematisch: $w = \phi \cdot l$

Constitutief: $M_1 = c \cdot 2\phi$

$M_2 = c \cdot \phi$

$$F \cdot w = M_1 + V \cdot l$$
$$F \cdot w = M_1 + M_2 - V \cdot l$$
$$\overline{2F \cdot w = 2M_1 + M_2}$$
$$2F \cdot \phi \cdot l = 2c \cdot 2\phi + c \cdot \phi$$
$$2F \cdot \phi \cdot l = 5c \cdot \phi \quad \rightarrow \quad \boxed{F = \frac{5}{2} \cdot \frac{c}{l}}$$

Resultaten matrix geometrisch en materiaal / lineair en niet-lineair

<u>Geometrisch lineair / Materiaal lineair</u>

$$F = \frac{5}{2} \cdot \frac{c}{l}$$

<u>Geometrisch lineair / Materiaal niet-lineair</u>

Evenwicht: $F_k \cdot w_i = M$

Constitutief: $M = M_p$

$$\left.\begin{array}{l} F_k \cdot w_i = M \\ M = M_p \end{array}\right\} F_k = \frac{M_p}{w_i} \qquad F_k = \frac{5}{2} \cdot \frac{c}{l}$$

$$\frac{M_p}{w_i} = \frac{5c}{2l} \qquad w_i = \frac{2M_p l}{5c} \qquad F = \frac{M_p}{w}$$

<u>Geometrisch niet-lineair / Materiaal lineair</u>

Evenwicht: $F \cdot w = M$

Kinematisch $w = \phi \cdot l$ en $w_0 = \phi_0 \cdot l$

Constitutief: $M = 2\,{}^1\!/_2\, c \cdot \phi$

$$F \cdot w = M = 2\,{}^1\!/_2\, c \cdot \phi \qquad w = l \cdot (\phi - \phi_0)$$

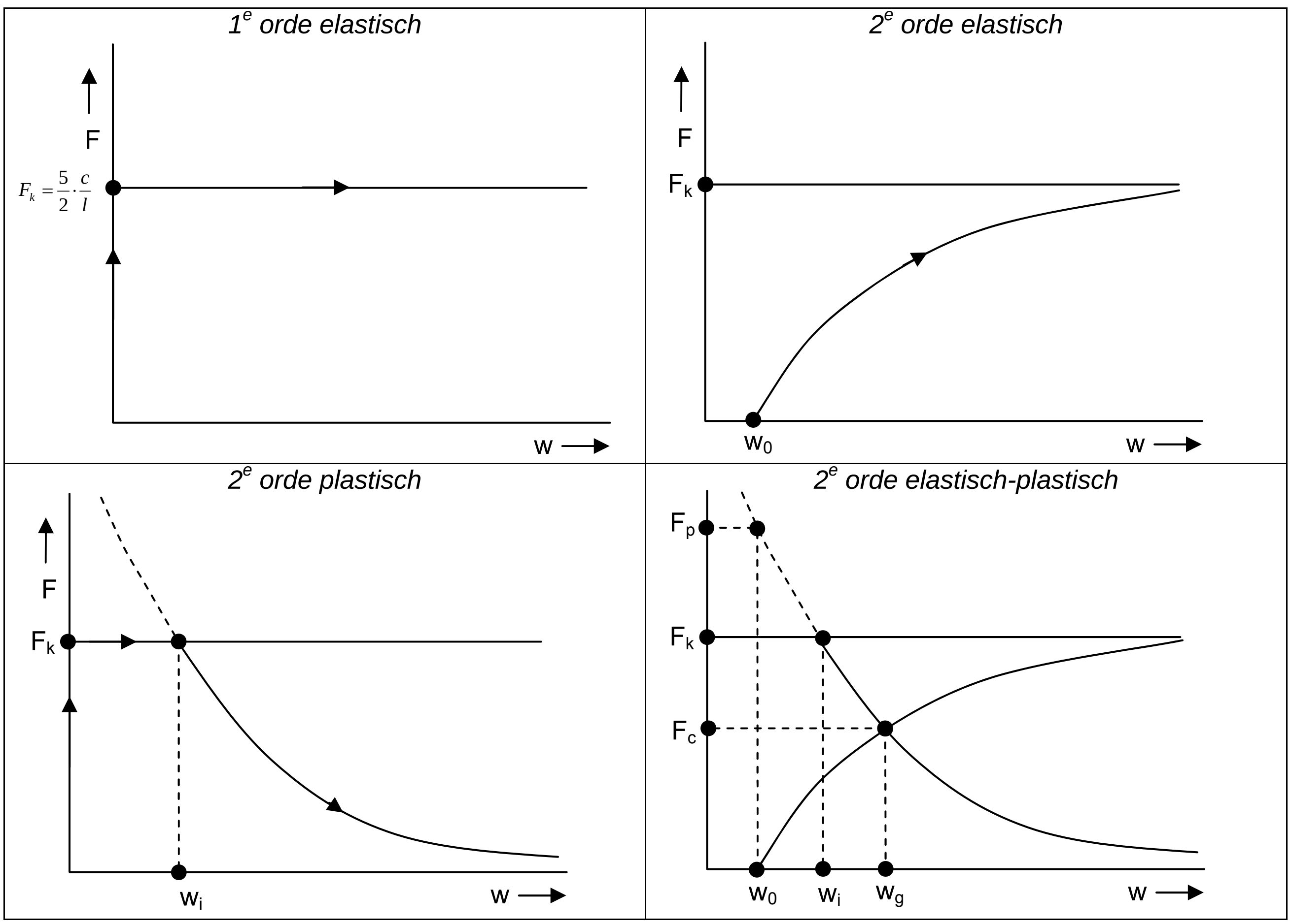
$$F \cdot \phi \cdot l = 2\,{}^1\!/_2\, c \cdot \phi \quad \rightarrow \quad F = \frac{2\,{}^1\!/_2\, c}{l}\left(1 - \frac{\phi_0}{\phi}\right) \quad \rightarrow \quad F_k = \left(1 - \frac{w_0}{w}\right)$$

Geometrisch niet-lineair / Materiaal niet-lineair

$F_c \cdot w_g = M = M_p$ of $w = w_g$ $\qquad F_c = \left(F_k \left(1 - \frac{w_0}{w_g} \right) \right)$

$$w_g = w_0 + \frac{5 \cdot M_p \cdot l}{2 \cdot c} = w_0 + \frac{M_p}{F_k} = w_0 + w_i$$

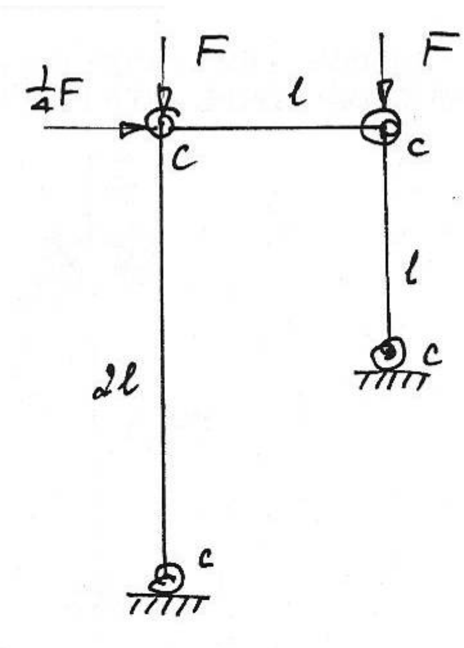
Merchant-Ranking: $\frac{1}{F_c} = \frac{1}{F_p} + \frac{1}{F_k}$ → $F_p = \frac{M_p}{w_0}$



Opdracht 2

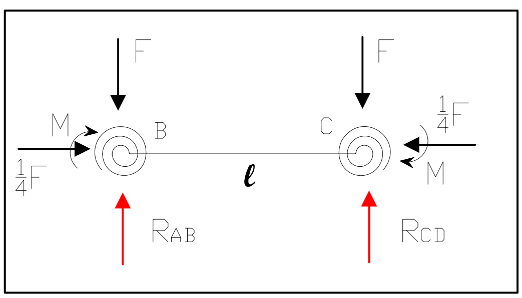
Opdrachtformulering

- Bepaal met behulp van de evenwichtsmethode en potentiële energie de kniklast.
  Zet de resultaten uit in een matrix geometrisch en materiaal / lineair en niet-lineair.



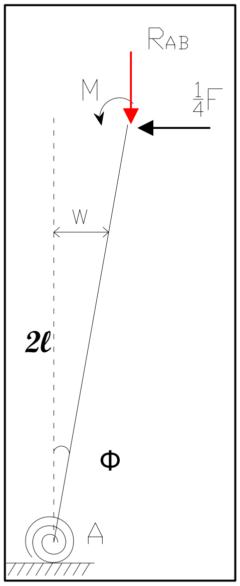
Uitwerking

Kniklast met behulp van evenwichtsmethode



$$\sum M_B = 0$$
$$M + M + F \cdot l - R_C \cdot l = 0$$
$$R_C = \frac{F \cdot l + 2M}{l} = F + \frac{2M}{l}$$

$$\sum M_C = 0$$
$$M + M - F \cdot l + R_B \cdot l = 0$$
$$R_B = \frac{F \cdot l - 2M}{l} = F - \frac{2M}{l}$$



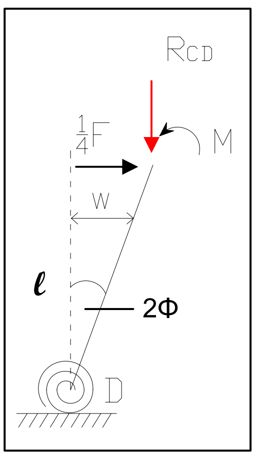
$$\sum M_A = 0$$

$$R_A \cdot w - \tfrac{1}{4} F \cdot 2l - M - M = 0$$

$$\left(F - \frac{2M}{l} \right) \cdot \phi \cdot 2l - \tfrac{2}{4} F \cdot l - 2M = 0$$

$$w = \phi \cdot 2l$$

$$R_{AB} = F - \frac{2M}{l}$$



$$\sum M_D = 0$$

$$R_{CD} \cdot w + \tfrac{1}{4} F \cdot l + M + M = 0$$

$$\left(F + \frac{2M}{l} \right) \cdot 2\phi \cdot l + \tfrac{1}{4} F \cdot l + 2M = 0$$

$$w = 2\phi \cdot l$$

$$R_{CD} = F + \frac{2M}{l}$$

$$\left(F - \frac{2M}{l} \right) \cdot \phi \cdot 2l - \tfrac{2}{4} F \cdot l - 2M = 0 \quad \rightarrow \quad 2F \cdot l \cdot \phi - 4M \cdot \phi - \tfrac{2}{4} F \cdot l - 2M = 0$$

$$\left(F + \frac{2M}{l} \right) \cdot 2\phi \cdot l + \tfrac{1}{4} F \cdot l + 2M = 0 \quad \rightarrow \quad 2F \cdot l \cdot \phi + 4M \cdot \phi + \tfrac{2}{4} F \cdot l + 2M = 0$$

$$\boxed{6F \cdot l \cdot \phi + 4M \cdot \phi + 6M = 0}$$

Constitutief

$$M = c \cdot \phi \rightarrow 6F \cdot l \cdot \phi + 4 \cdot c \cdot \phi^2 + 6 \cdot c \cdot \phi = 0 \Rightarrow F = \frac{10c\phi}{6l\phi}$$

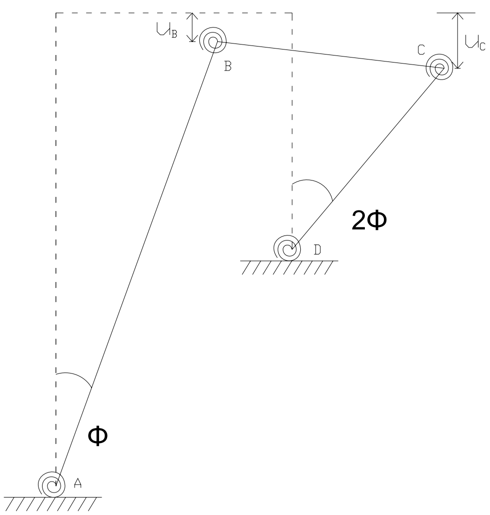
$$\phi = klein \Rightarrow F = \frac{10c}{6l} = \frac{5c}{3l}$$

Kniklast met behulp van potentiële energie

Inwendige potentiële energie

$$E_{pot;i} = \frac{1}{2}c(2\phi)^2 + \frac{1}{2}c(2\phi)^2 + \frac{1}{2}c(\phi)^2 + \frac{1}{2}c(\phi)^2$$
$$= 5C(\phi)^2$$

Uitwendige potentiële energie



$$u_B = 2l - 2l\cos\phi$$
$$u_C = l - l\cos 2\phi$$

$$E_{pot;u} = -Fu_B - Fu_C$$
$$E_{pot;u} = -F\left(2l - l\cos\phi\right) - F\left(l - l\cos\phi\right)$$
$$= -F\left(2l - \left(2l(1-\frac{1}{2}\phi^2\right)\right) - F\left(l - \left(l(1-\frac{1}{2}(2\phi)^2\right)\right)$$
$$= -Fl\phi^2 - 2Fl\phi^2 = -3Fl\phi^2$$

$$\cos\phi = 1 - \frac{1}{2}\phi^2$$
$$\cos 2\phi = 1 - \frac{1}{2}(2\phi)^2$$

Totale potentiële energie

$$E_{pot;tot} = E_{pot;i} + E_{pot;u} = 5C\phi^2 - 3Fl\phi^2$$

$$\delta E_{pot} = 0 \quad \rightarrow \quad \frac{\delta E_{pot}}{\delta\phi} = 0$$

$$\delta E_{pot} = 5c\phi^2 - 3Fl\phi^2 = 0 \quad \rightarrow \quad \phi^2 \neq 0$$

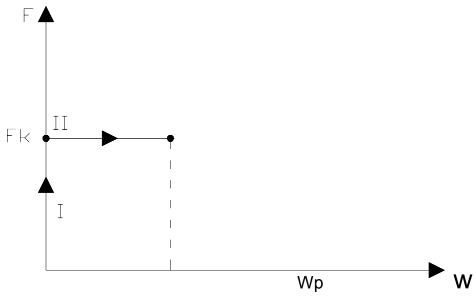
$$\delta E_{pot} = \left(5c - 3Fl\right)\phi^2 = 0$$
$$\left(5c - 3Fl\right) = 0 \quad \rightarrow \quad \boxed{F = \frac{5c}{3l}}$$

Resultaten matrix geometrisch en materiaal / lineair en niet-lineair

Geometrisch lineair – materiaal lineair

$F = \frac{5c}{3l}$ $\qquad l_k = \frac{5c}{3F}$

I - $F<F_k$ stabiel
II - $F>F_k$ instabiel

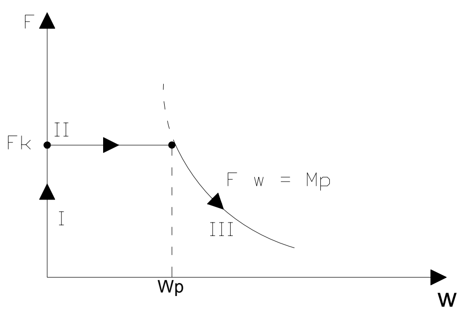


Geometrisch lineair - materiaal niet-lineair

$M = M_p$

$$\left.\begin{array}{l} Evenwicht: F_k \cdot w = M \\ Constitutief: M = M_p \end{array}\right] F_k = \frac{M_p}{w}$$

$$F_k = \frac{5c}{3l} \Rightarrow \frac{5c}{3l} = \frac{M_p}{w} \Rightarrow w = \frac{M_p \cdot 3l}{5c} = \frac{3}{5}\frac{M_p \cdot l}{c}$$



Geometrisch niet-lineair – materiaal lineair

$n = \frac{F_k}{F}$

$w = \frac{n}{n-1} w_0$

Geometrisch niet-lineair – materiaal lineair

Eerste orde evenwicht

$$\tfrac{1}{4}F_H \cdot 2l = 4M$$

Constitutief

$$M = c \cdot \phi_0$$

$$\phi_0 = \frac{F_H \cdot l}{8c}$$

Tweede orde evenwicht

$$\left(F - \frac{2M}{l}\right) \cdot \phi \cdot 2l + \tfrac{1}{4}F_H \cdot l - H \cdot l - 2M = 0$$

$$\left(F + \frac{2M}{l}\right) \cdot 2\phi \cdot l + H \cdot l - 2M = 0 \quad +$$

$$4F \cdot l \cdot \phi + \tfrac{1}{4}F_H \cdot l - 4M = 0$$

$$M = c \cdot \phi$$

$$4F \cdot l \cdot \phi + \tfrac{1}{4}F_H \cdot l - 4c \cdot \phi = 0 \quad \times 4$$

$$8F \cdot l \cdot \phi + F_H \cdot l - 8c \cdot \phi = 0$$

$$\phi(8c - 8F \cdot l) = F_H \cdot l$$

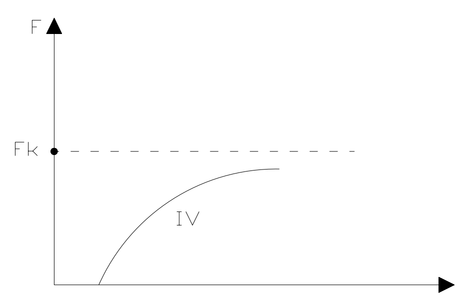
$$\phi = \frac{F_H \cdot l}{8c - 8F \cdot l} = \frac{F_H \cdot l}{8c}\left(\frac{1}{1 - F\frac{l}{c}}\right)$$

$$\phi_0 = \frac{F_H \cdot l}{8c}; F_k = \frac{5c}{3l} \Rightarrow \frac{1}{F_k} = \frac{3l}{5c}$$

$$\phi = \phi_0 \cdot \left(\frac{1}{1 - \frac{5F}{3F_k}}\right)$$

$$n = \frac{F}{F_k}$$

$$\phi = \phi_0 \cdot \left(\frac{1}{1 - \frac{5F}{3F_k}}\right) = \tfrac{5}{3} \cdot \frac{n}{n-1} \cdot \phi_0$$



Geometrisch niet lineair – materiaal niet lineair

$$M = M_p$$

$$M_p = \phi_p \cdot c$$

$$\left(F_c - \frac{2M_p}{l}\right) \cdot \phi_p \cdot 2l + \tfrac{1}{4} F_H \cdot l - H \cdot l - 2M_p = 0$$

$$\left(F_c + \frac{2M_p}{l}\right) \cdot 2\phi_p \cdot l + H \cdot l - 2M_p = 0 \quad +$$

$$4F_c \cdot \phi_p \cdot l + \tfrac{1}{4} F_H \cdot l - 4M_p = 0 \quad \times 4$$

$$16F_c \cdot \phi_p \cdot l + F_H \cdot l - 16M_p = 0$$

$$\phi_p = \frac{M_p}{c}; F_H = \alpha \cdot F_c$$

$$F_c\left(16 \cdot \phi_p \cdot l + \frac{F_H \cdot l}{F_c}\right) = 16M_p$$

$$16\phi_p \cdot l + \frac{\alpha F_H l}{F_c} = \frac{16M_p}{F_c}$$

$$16\frac{M_p}{c} \cdot l + \frac{\alpha \cdot F_c l}{F_c} = \frac{16M_p}{F_c}$$

$$\frac{l}{c} + \frac{\alpha \cdot l}{16M_p} = \frac{1}{F_c}$$

$$F_k = \frac{5c}{3l} \Rightarrow \frac{1}{F_k} = \frac{3l}{5c}$$

$$\alpha \cdot F_p \cdot 2l = 4M_p$$

$$F_p = \frac{2M_p}{\alpha \cdot l} \Rightarrow \frac{1}{F_p} = \frac{\alpha \cdot l}{2M_p}$$

$$Merchant - Ranking$$

$$\frac{1}{F_c} = \frac{1}{F_p} + \frac{1}{F_k}$$

$$\phi_0 = \frac{F_H \cdot l}{16c}; \alpha \cdot F = \frac{2c}{l} \cdot \phi$$

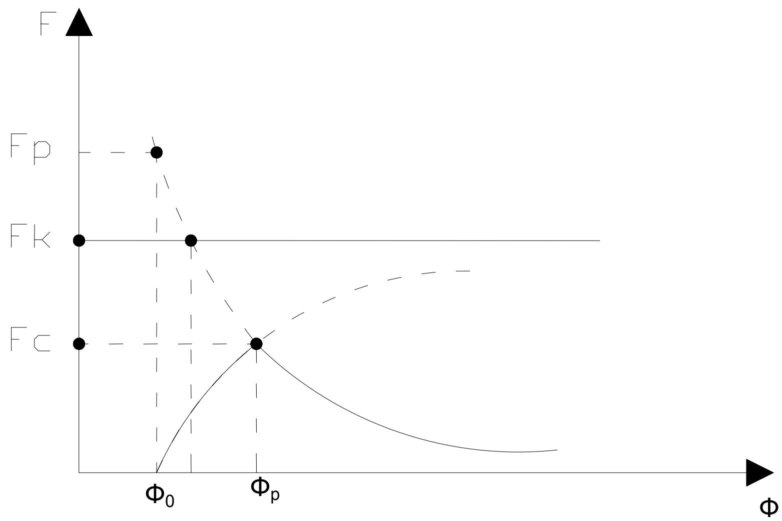
$$F_p = \frac{2M_p}{\alpha \cdot l}$$

$$F = \frac{1}{\phi_0 + \phi}\frac{2M_p}{l}; \phi = 0$$

$$F = \frac{1}{\phi_0}\frac{2M_p}{l} \Rightarrow F = \frac{8c}{F_H \cdot l}\frac{2M_p}{l}$$

$$M_p = M_p\left(1-\frac{F^2}{F_p^2}\right); F_p = F_p\left(1-\frac{F^2}{F_p^2}\right)$$

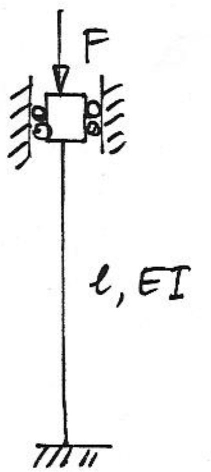
$$\frac{1}{F_c} = \frac{1}{F_k} + \frac{1}{F_p\left(1-\frac{F_c^2}{F_p^2}\right)} \Rightarrow \frac{1}{F_c} = \frac{1}{F_k} + \frac{1}{\frac{2M_p}{\alpha\cdot l}\left(1-\frac{F_c^2}{\frac{2M_p}{\alpha\cdot l}}\right)}$$



Opdracht 3

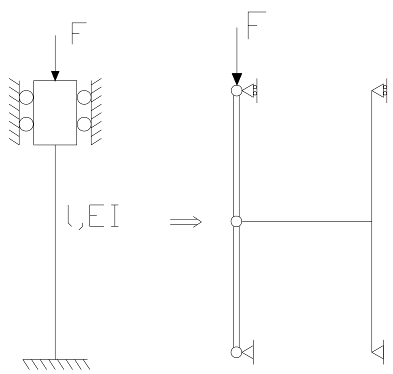
<u>Opdrachtformulering</u>

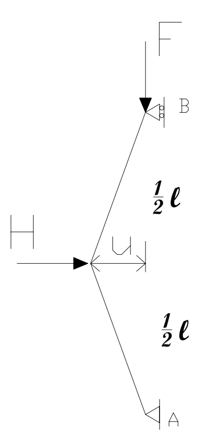
- Bepaal de kniklast met behulp van de pendelmethode en potentiële energie.
- Geef conclusies over de nauwkeurigheid van de berekende resultaten: bovengrens of ondergrens



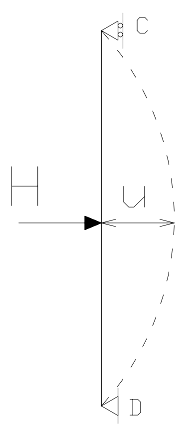
Uitwerking

Kniklast met behulp van pendelmethode

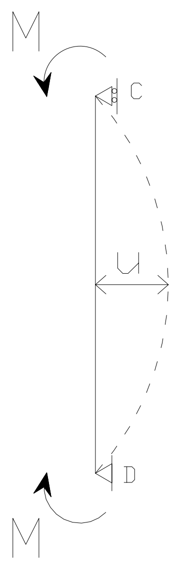




$$F \cdot u - \tfrac{1}{2} l \cdot \tfrac{1}{2} H = 0$$
$$F \cdot u - \tfrac{1}{4} H \cdot l = 0$$



$$u = \frac{H \cdot l^3}{48EI}$$
$$\varphi_B = \frac{H \cdot \tfrac{1}{2} l \cdot \tfrac{1}{2} l \cdot \tfrac{3}{2} l}{6EI \cdot l}$$
$$\varphi_B = \frac{\tfrac{3}{8} H \cdot l^3}{6EI \cdot l} = \frac{H \cdot l^2}{16EI}$$



$$\varphi_C = -\frac{Ml}{3EI} - \frac{Ml}{6EI} = -\frac{Ml}{2EI}$$

$$\varphi_B = \varphi_C$$

$$\frac{H \cdot l^2}{16EI} = -\frac{M \cdot l}{2EI}$$

$$-M \cdot l \cdot 16EI = H \cdot l^2 \cdot 2EI$$

$$M = -\frac{H \cdot l}{8}$$

$$u_{CD} = 2 \cdot \frac{Ml^2}{16EI} = \frac{Ml^2}{8EI}$$

$$M = \frac{8EI}{l^2} \cdot u$$

$$-\frac{H \cdot l}{8} = \frac{8EI}{l^2} \cdot u$$

$$u = -\frac{H \cdot l^3}{64EI}$$

$$u = \frac{H \cdot l^3}{48EI} - \frac{H \cdot l^3}{64EI} = \frac{H \cdot l^3}{192EI}$$

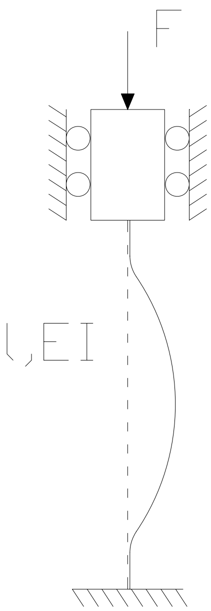
$$F \cdot u - \tfrac{1}{4} H \cdot l = 0$$

$$F \cdot \frac{H \cdot l^3}{192EI} - \frac{H \cdot l}{4} = 0$$

$$F = \frac{192EI \cdot H \cdot l}{4H \cdot l^3}$$

$$F_k = \frac{48EI}{l^2}$$

Kniklast met behulp van potentiële energie



probeerfunctie :

$$w(x) = a + bx + cx^2 + dx^3 + ex^4$$

$$\varphi(x) = \frac{dw}{dx} = b + 2cx + 3dx^2 + 4ex^3$$

$$\kappa(x) = \frac{d^2w}{dx^2} = 2c + 6dx + 12ex^2$$

$$x = 0 \Rightarrow w(0) = 0 \rightarrow a = 0$$

$$x = 0 \Rightarrow \varphi(0) = 0 \rightarrow b = 0$$

$$x = l \Rightarrow w(l) = 0 \rightarrow cl^2 + dl^3 + el^4 = 0$$

$$x = l \Rightarrow \varphi(l) = 0 \rightarrow 2cl + 3dl^2 + 4el^3 = 0$$

d en e uitdrukken in c

$$\begin{array}{l} cl^2 + dl^3 + el^4 = 0 \\ 2cl + 3dl^2 + 4el^3 = 0 \end{array} \left| \begin{array}{l} \times \frac{4}{l} \\ \times 1 \end{array} \right| \begin{array}{l} 4cl + 4dl^2 + 4el^3 = 0 \\ 2cl + 3dl^2 + 4el^3 = 0 \_ \end{array}$$

$$2cl + dl^2 = 0$$

$$d = -\frac{2c}{l}$$

$$\begin{array}{l} cl^2 + dl^3 + el^4 = 0 \\ 2cl + 3dl^2 + 4el^3 = 0 \end{array} \left| \begin{array}{l} \times \frac{3}{l} \\ \times 1 \end{array} \right| \begin{array}{l} 3cl + 3dl^2 + 3el^3 = 0 \\ 2cl + 3dl^2 + 4el^3 = 0 \_ \end{array}$$

$$cl - el^3 = 0$$

$$e = \frac{c}{l^2}$$

$$w(x) = cx^2 + dx^3 + ex^4 = cx^2 - \frac{2c}{l}x^3 + \frac{c}{l^2}x^4 \Rightarrow c\left(x^2 - \frac{2x^3}{l} + \frac{x^4}{l^2}\right)$$

$$\varphi(x) = \frac{dw}{dx} = 2cx + 3dx^2 + 4ex^3 = 2cx - \frac{6c}{l}x^2 + \frac{4c}{l^2}x^3 \Rightarrow c\left(2x - \frac{6x^2}{l} + \frac{4x^3}{l^2}\right)$$

$$\kappa(x) = \frac{d^2w}{dx^2} = 2c + 6dx + 12ex^2 = 2c - \frac{12c}{l}x + \frac{12c}{l^2}x^2 \Rightarrow c\left(2 - \frac{12x}{l} + \frac{12x^2}{l^2}\right)$$

Inwendige potentiële energie

$$E_{pot;i} = \tfrac{1}{2} EI \int_0^l (\kappa(x))^2 dx$$

$$E_{pot;i} = \tfrac{1}{2} EI \int_0^l \left(c(2 - \frac{12x}{l} + \frac{12x^2}{l^2}) \right)^2 dx$$

$$E_{pot;i} = \tfrac{1}{2} EI \int_0^l c^2 \cdot (2 - \frac{12x}{l} + \frac{12x^2}{l^2})^2 dx$$

$$E_{pot;i} = \tfrac{1}{2} EI \cdot c^2 \int_0^l (2 - \frac{12x}{l} + \frac{12x^2}{l^2})^2 dx$$

$$\left[(2 - \frac{12x}{l} + \frac{12x^2}{l^2})^2 = (2 - \frac{12x}{l} + \frac{12x^2}{l^2}) \times (2 - \frac{12x}{l} + \frac{12x^2}{l^2}) = 4 - \frac{48x}{l} + \frac{192x^2}{l^2} - \frac{288x^3}{l^3} + \frac{144x^4}{l^4} \right]$$

$$E_{pot;i} = \tfrac{1}{2} EI \cdot c^2 \int_0^l 4 - \frac{48x}{l} + \frac{192x^2}{l^2} - \frac{288x^3}{l^3} + \frac{144x^4}{l^4} dx$$

$$= \tfrac{1}{2} EI \cdot c^2 \left[4x - \frac{24x^2}{l} + \frac{64x^3}{l^2} - \frac{72x^4}{l^3} + \frac{144x^5}{5l^4} \right]_0^l$$

$$= \tfrac{1}{2} EI \cdot c^2 \left[4l - \frac{24l^2}{l} + \frac{64l^3}{l^2} - \frac{72l^4}{l^3} + \frac{144l^5}{5l^4} \right] = \tfrac{1}{2} EI \cdot c^2 [4l - 24l + 64l - 72l + 144l]$$

$$= \tfrac{1}{2} EI \cdot c^2 \cdot \tfrac{4}{5} l$$

$$E_{pot;i} = \tfrac{2}{5} \cdot c^2 \cdot l \cdot EI$$

Uitwendige potentiële energie

$$E_{pot;u} = -\tfrac{1}{2} F \int_0^l (\varphi(x))^2 dx$$

$$E_{pot;u} = -\tfrac{1}{2} F \int_0^l c \left(2x - \frac{6x^2}{l} + \frac{4x^3}{l^2} \right)^2 dx$$

$$E_{pot;u} = -\tfrac{1}{2} F \int_0^l c^2 \cdot (2x - \frac{6x^2}{l} + \frac{4x^3}{l^2})^2 dx$$

$$E_{pot;u} = -\tfrac{1}{2} F \cdot c^2 \int_0^l (2x - \frac{6x^2}{l} + \frac{4x^3}{l^2})^2 dx$$

$$\left[(2x - \frac{6x^2}{l} + \frac{4x^3}{l^2})^2 = (2x - \frac{6x^2}{l} + \frac{4x^3}{l^2}) \times (2x - \frac{6x^2}{l} + \frac{4x^3}{l^2}) = 4x^2 - \frac{24x^3}{l} + \frac{52x^4}{l^2} - \frac{48x^5}{l^3} + \frac{16x^6}{l^4} \right]$$

$$E_{pot;u} = -\tfrac{1}{2} F \cdot c^2 \int_0^l 4x^2 - \frac{24x^3}{l} + \frac{52x^4}{l^2} - \frac{48x^5}{l^3} + \frac{16x^6}{l^4} dx$$

$$= -\tfrac{1}{2} F \cdot c^2 \left[\tfrac{4}{3} x^3 - \frac{6x^4}{l} + \frac{52x^5}{5l^2} - \frac{8x^6}{l^3} + \frac{16x^7}{7l^4} \right]_0^l$$

$$= -\tfrac{1}{2} F \cdot c^2 \left[\tfrac{4}{3} l^3 - \frac{6l^4}{l} + \frac{52l^5}{5l^2} - \frac{8l^6}{l^3} + \frac{16l^7}{7l^4} \right] = -\tfrac{1}{2} F \cdot c^2 \left[\tfrac{4}{3} l^3 - 6l^3 + \tfrac{52}{5} l^3 - 8l^3 + \tfrac{16}{7} l^3 \right]$$

$$= -\tfrac{1}{2} F \cdot c^2 \cdot \tfrac{2}{105} l$$

$$E_{pot;u} = -\tfrac{1}{105} \cdot F \cdot c^2 \cdot l^3$$

Totale potentiële energie

$$E_{pot;tot} = E_{pot;i} + E_{pot;u}$$

$$E_{pot;tot} = \tfrac{2}{5} EI \cdot c^2 \cdot l - \tfrac{1}{105} F \cdot c^2 \cdot l^3$$

$$\frac{\partial E_{pot}}{\partial c^2} = 0$$

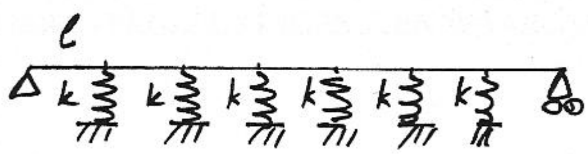
$$c^2 \neq 0 \Rightarrow \tfrac{2}{5} EI \cdot l - \tfrac{1}{105} F \cdot l^3 = 0$$

$$F_k = \frac{\tfrac{2}{5} EI \cdot l}{\tfrac{1}{105} l^3} = \frac{42EI}{l^2}$$

Opdracht 4

Opdrachtformulering

- Bepaal de kritische kniklengte



Uitwerking

Beddingsconstante: $k = \dfrac{6c}{l}$

Engesser: $2\sqrt{kEI} = \dfrac{\pi^2 EI}{l_k^{\,2}}$ → $l_k = \pi \sqrt[4]{\dfrac{4EI}{k}}$

vervang $k = \dfrac{6c}{l}$ in de formule $l_k = \pi \sqrt[4]{\dfrac{4EI}{k}}$

$$l_k = \pi \sqrt[4]{\dfrac{4EI}{\dfrac{6c}{l}}}$$

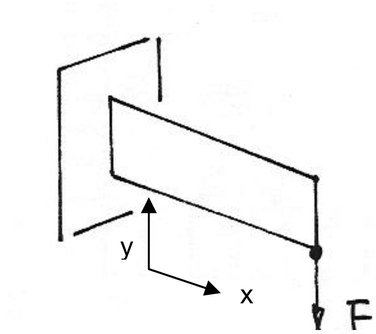
$$l_k = \pi \sqrt[4]{\dfrac{4EI \cdot l}{6c}}$$

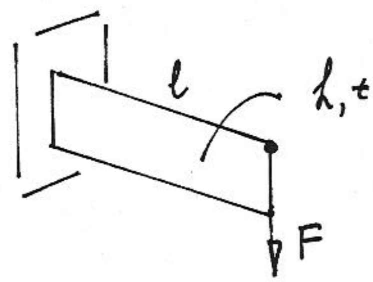
$$\boxed{l_k = \pi \sqrt[4]{\dfrac{2EI \cdot l}{3c}}}$$

Opdracht 5

Opdrachtformulering

- Stel differentiaalvergelijkingen op voor kip voor de onderstaande twee gevallen.
- Wat is vanuit het oogpunt de hoogste kiplast het gewenste aangrijpingspunt. Laat in de opgestelde differentiaalvergelijkingen zijn waarom.





Uitwerking

Randvoorwaarde

$M_y = F(l - x)$

Buiging

$M = -\frac{d^2 v}{dx^2} EI_y$ $\qquad$ $v = \frac{1}{2} h\varphi$

$M = -EI_{fl} * \frac{1}{2} h \frac{d^2\varphi}{dx^2}$ $\qquad$ $v_y = \frac{dM}{dx} = -EI_{fl} \frac{1}{2} h \frac{d^3\varphi}{dx^3}$

Torsie

Er treedt geen welving op $\qquad$ $\frac{d\varphi}{dx} = \frac{T}{GI_{wr}}$ = CONSTANT (pure torsie)

$T = GI_{wr} \frac{d\varphi}{dx}$

Buiging horizontaal: $\qquad$ $v = \frac{dV}{dx}$

Buiging verticaal: $\qquad$ $\psi = -\frac{dw}{dz}$

Kleine vervormingen, kleine hoekverplaatsingen

$\cos v \approx 1$ $\qquad$ $\cos\varphi \approx 1$

$\sin v \approx v$ $\qquad$ $\sin\varphi \approx 1$

Dan geldt,

$$M_\eta = EI_y \frac{d^2 w}{dx^2} - M_y = 0$$

$$M_\xi = -EI_z \frac{d^2 v}{dx^2} + \varphi M_y = 0$$

Invloed tweede orde effect:

a) De kracht aan de onderzijde van de balk

$$M_\zeta = +GI_{wr} \frac{d\varphi}{dx} + M_y \frac{dv}{dx} = 0$$

Uitwerking:

$$+GI_{wr} \frac{d\varphi}{dx} + F(l-x)\frac{dv}{dx} = 0 \qquad M_y = F(l-x)$$

$$+GI_{wr} \frac{d^2\varphi}{dx^2} - Fl\frac{dv}{dx} + F(l-x)\frac{d^2 v}{dx^2} = 0$$ → Eerste differentiatie

$$+GI_{wr} \frac{d^3\varphi}{dx^3} - Fl\frac{d^2 v}{dx^2} - Fl\frac{d^2 v}{dx^2} + F(l-x)\frac{d^3 v}{dx^3} = 0$$ → Tweede differentiatie

$$+GI_{wr} \frac{d^3\varphi}{dx^3} - 2Fl\frac{d^2 v}{dx^2} + F(l-x)\frac{d^3 v}{dx^3} = 0$$

b) De kracht bovenop de balk

$$M_\zeta = +GI_{wr} \frac{d\varphi}{dx} - M_y \frac{dv}{dx} = 0$$

Uitwerking:

$$+GI_{wr} \frac{d^3\varphi}{dx^3} + Fl\frac{d^2 v}{dx^2} + Fl\frac{d^2 v}{dx^2} - F(l-x)\frac{d^3 v}{dx^3} = 0$$ → Zelfde stappen

$$+GI_{wr} \frac{d^3\varphi}{dx^3} + 2Fl\frac{d^2 v}{dx^2} - F(l-x)\frac{d^3 v}{dx^3} = 0$$

Conclusie

Zoals in onderstaande afleidingen is af te lezen, zijn de sterkte eigenschappen (GI) van situatie a) gelijk aan situatie b). De tweede term is afwijkend. De derde term valt (bij invullen van x = l) weg bij situatie b). Bij situatie a) wordt deze term twee maal zo groot op x=l.

a) $$+GI_{wr}\frac{d^3\varphi}{dx^3}-2Fl\frac{d^2v}{dx^2}+F(l-x)\frac{d^3v}{dx^3}=0$$

b) $$+GI_{wr}\frac{d^3\varphi}{dx^3}+2Fl\frac{d^2v}{dx^2}-F(l-x)\frac{d^3v}{dx^3}=0$$

Ad a) x = L : $$+GI_{wr}\frac{d^3\varphi}{dx^3}-2Fl\frac{d^2v}{dx^2}=0$$

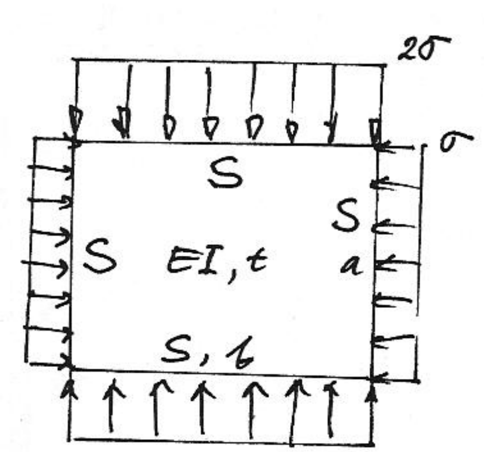
Ad b) x = L: $$+GI_{wr}\frac{d^3\varphi}{dx^3}+2Fl\frac{d^2v}{dx^2}=0$$

Situatie a) is veel gunstig voor het opnemen van de kiplast. De kracht vermenigvuldigd met de afgeleide van de hoekverdraaiing (φ) zorgt voor een vermindering van de opneembare kiplast. De tweede afgeleide van de hoekverdraaiing (φ) zorgt juist voor een sterke vergroting van de kiplast. Het is daarom wenselijk zoveel mogelijk te ontwerpen volgens het principe dat is aangegeven in situatie a).

Opdracht 6

Opdrachtformulering

- Bepaal de kritische knikspanning voor verschillende waarden van lengte en breedte.



Uitwerking

$$-K\left(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}\right)+\sigma_x t\left(\frac{\partial^2 w}{\partial x^2}\right)+\sigma_y t\left(\frac{\partial^2 w}{\partial y^2}\right)=0$$

B.C.	x = 0 - x = a:	w = 0 - m_x = 0
	y = 0 - y = b:	w = 0 - m_y = 0

$$m_x=\frac{\partial^2 w}{\partial x^2}+v\frac{\partial^2 w}{\partial y^2}=0 \qquad m_y=v\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}=0$$

Oplossing; exact indien kinematische en dynamische randvoorwaarden voldoen:

$$w=C\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$ -> (=C*s*s)

$$\sigma_x\frac{t}{K}C\frac{\pi^2 m^2}{a^2}*s*s+\sigma_y\frac{t}{K}C\frac{\pi^2 n^2}{b^2}*s*s=C\left(\frac{\pi^4 m^4}{a^4}+2\frac{\pi^4 m^2 n^2}{a^2 b^2}+\frac{\pi^4 n^4}{b^4}\right)*s*s$$

$$\sigma_x\frac{t}{K}C\frac{\pi^2 m^2}{a^2}+\sigma_y\frac{t}{K}C\frac{\pi^2 n^2}{b^2}=C\left(\frac{\pi^4 m^4}{a^4}+2\frac{\pi^4 m^2 n^2}{a^2 b^2}+\frac{\pi^4 n^4}{b^4}\right) \qquad \boxed{\sigma_x=\sigma,\quad \sigma_y=2\sigma}$$

$$\sigma\frac{t}{K}C\frac{\pi^2 m^2}{a^2}+2\sigma\frac{t}{K}C\frac{\pi^2 n^2}{b^2}=C\left(\frac{\pi^4 m^4}{a^4}+2\frac{\pi^4 m^2 n^2}{a^2 b^2}+\frac{\pi^4 n^4}{b^4}\right)$$

$$\sigma\frac{t}{K}C\left(\frac{\pi^2 m^2}{a^2}+2\frac{\pi^2 n^2}{b^2}\right)=C\left(\frac{\pi^4 m^4}{a^4}+2\frac{\pi^4 m^2 n^2}{a^2 b^2}+\frac{\pi^4 n^4}{b^4}\right) \qquad C\neq 0$$

$$\sigma\frac{t}{K}\left(\frac{\pi^2m^2}{a^2}+2\frac{\pi^2n^2}{b^2}\right)-\left(\frac{\pi^2n^2}{a^2}+\frac{2\pi^2n^2}{b^2}\right)^2=0$$

$$S=\frac{\left(\frac{\pi^2 m^2}{a^2}+\frac{\pi^2 n^2}{b^2}\right)^2 K}{t\left(\frac{\pi^2 m^2}{a^2}+\frac{2\pi^2 n^2}{b^2}\right)}$$

Oplossing

$$S=\frac{(b^4m^4+2a^2b^2n^2m^2+n^4a^4)\pi^2K}{a^2b^2(m^2b^2+2n^2a^2)t}$$

$$\sigma=\frac{\pi^2K}{b^2t}*\frac{(m^2b^2+n^2a^2)^2}{a^2(m^2b^2+2n^2a^2)}=\frac{\pi^2K}{b^2t}*\alpha^2;\qquad \alpha^2=\frac{(m^2b^2+n^2a^2)^2}{a^2(m^2b^2+2n^2a^2)}$$

$$\alpha_{m,n}=\frac{m^2b^2+n^2a^2}{a\sqrt{m^2b^2+2n^2a^2}}$$

n=1

$$\alpha_{m,n=1}=\frac{m^2b^2+a^2}{a\sqrt{m^2b^2+2a^2}}$$

Stel m=1, n=1 $$\alpha_{m=1}=\frac{b^2+a^2}{a\sqrt{b^2+2a^2}}$$

Stel m=2, n=1 $$\alpha_{m=2}=\frac{4b^2+a^2}{a\sqrt{4b^2+2a^2}}$$

Stel m=3, n=1 $$\alpha_{m=3}=\frac{9b^2+a^2}{a\sqrt{9b^2+2a^2}}$$

n=2

$$\alpha_{m,n=1}=\frac{m^2b^2+4a^2}{a\sqrt{m^2b^2+8a^2}}$$

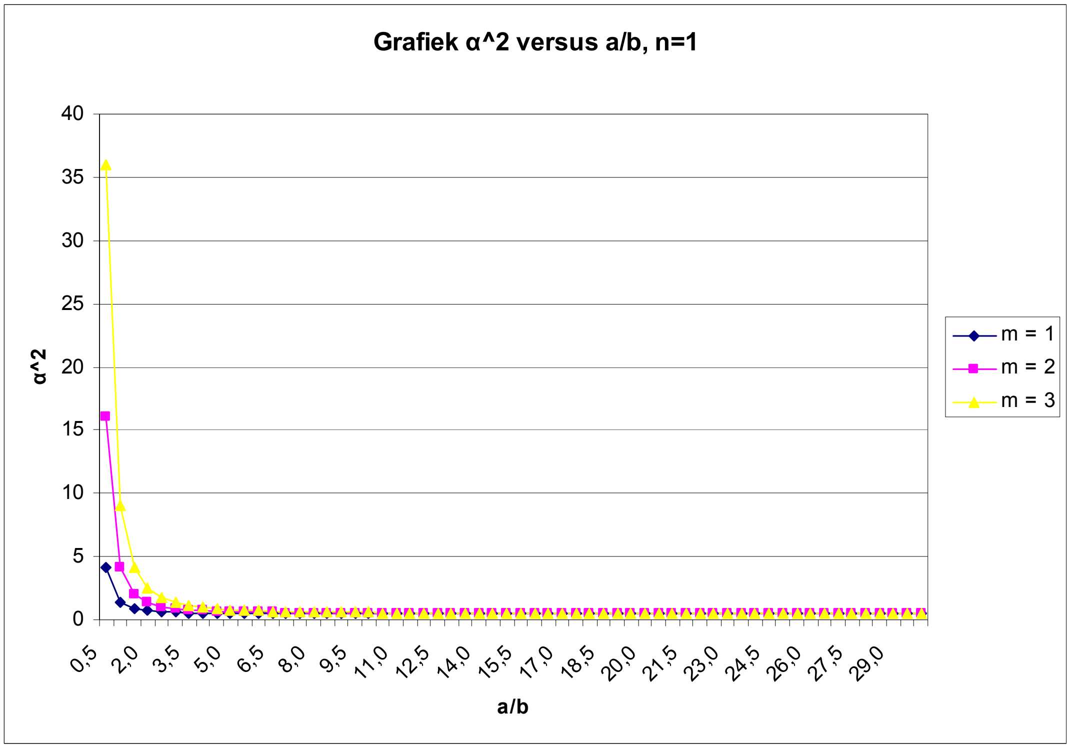
Stel m=1, n=2 $$\alpha_{m=1}=\frac{b^2+4a^2}{a\sqrt{b^2+8a^2}}$$

Stel m=2, n=2 $$\alpha_{m=2}=\frac{4b^2+4a^2}{a\sqrt{4b^2+8a^2}}$$

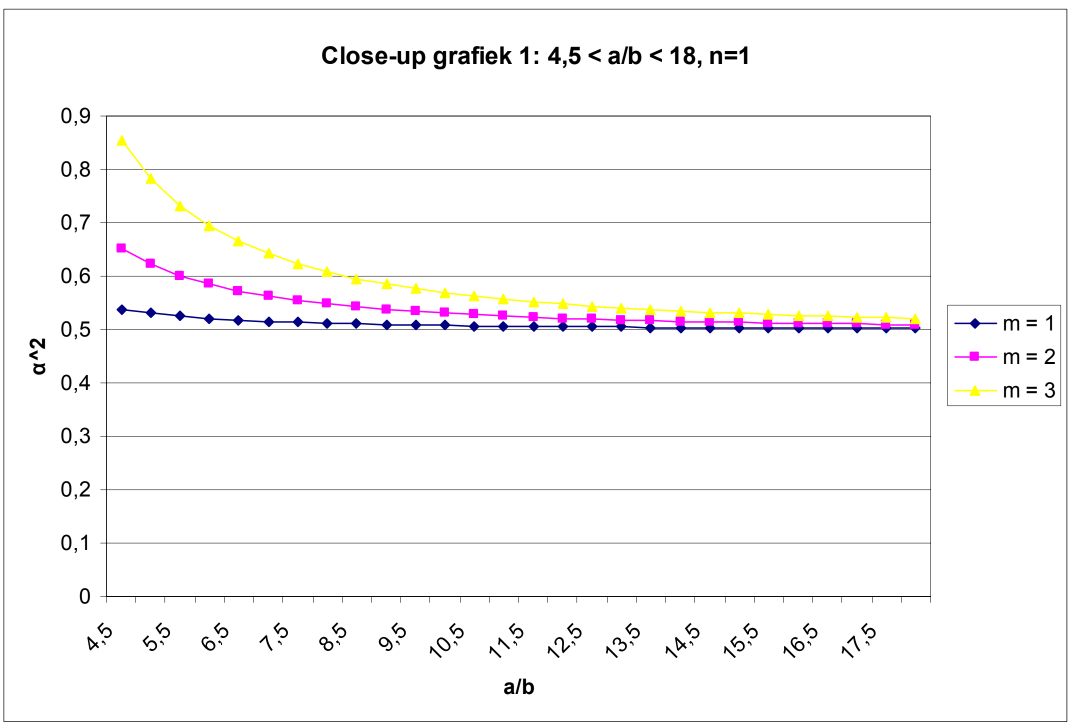
Stel m=3, n=2 $$\alpha_{m=3}=\frac{9b^2+4a^2}{a\sqrt{9b^2+8a^2}}$$

Grafieken

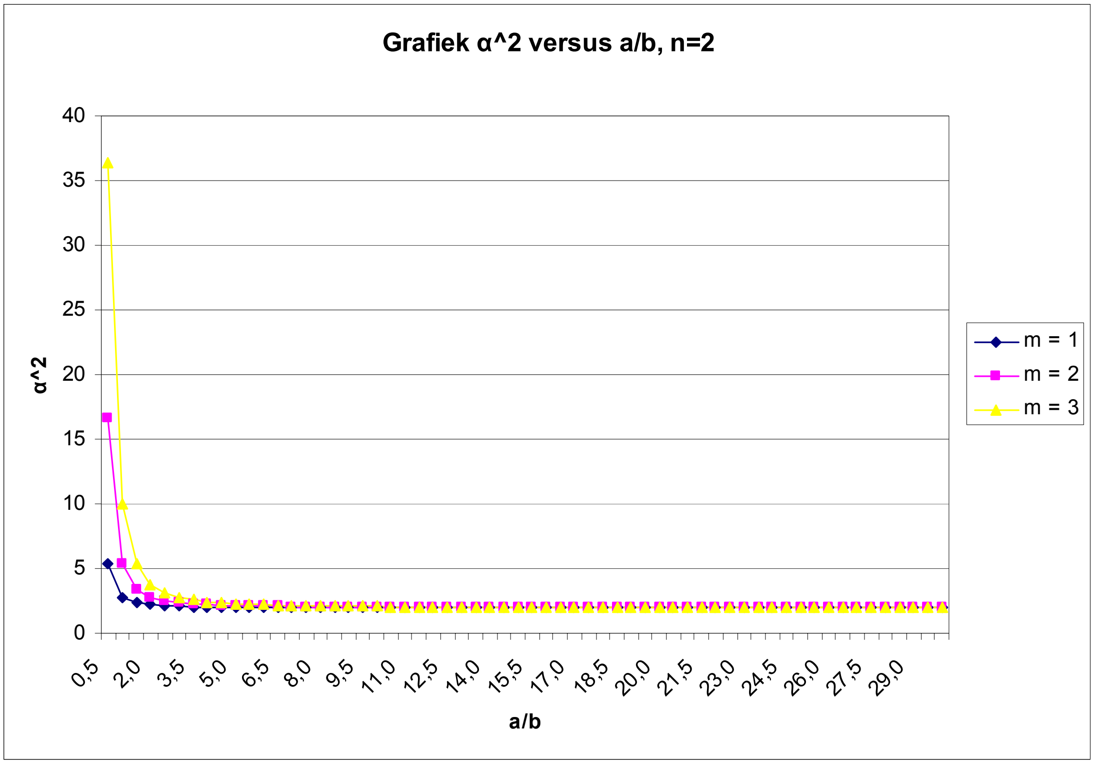
Invoer grafiek α^2 versus a/b met n=1:
n = 1, m = 1, 2, 3, 0,5 < a < 30, b = 1



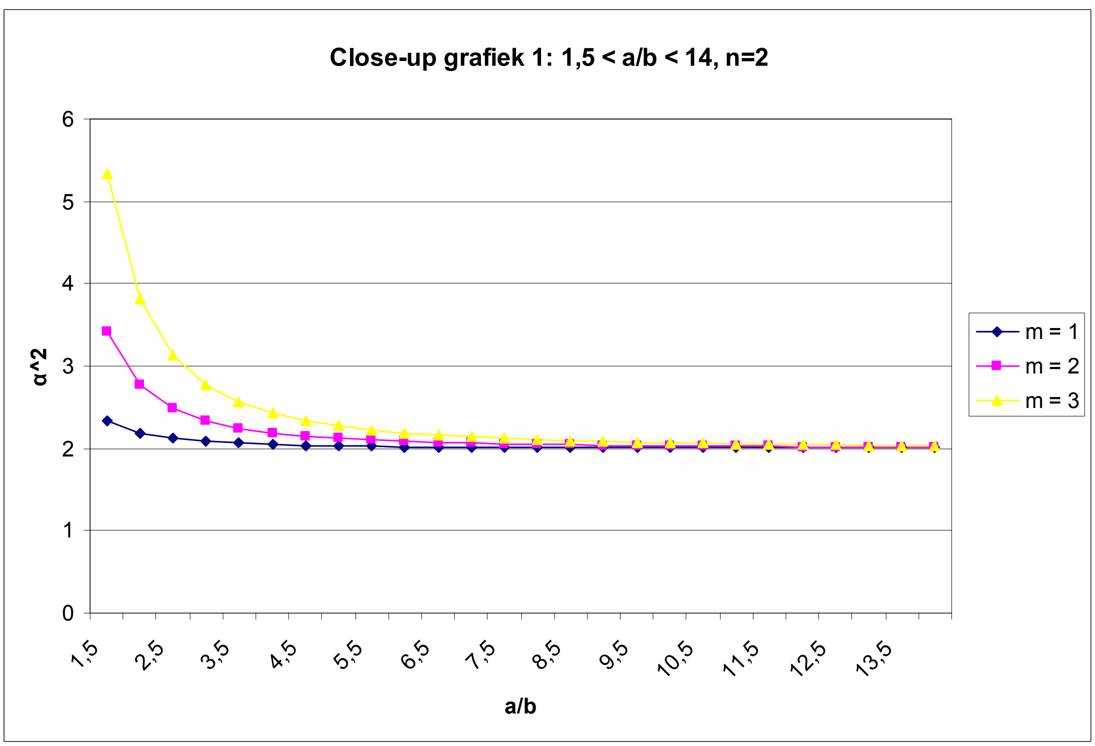
Invoer close-up grafiek:
4,5< a < 18



Invoer grafiek α^2 versus a/b met n=2:
n = 2, m = 1, 2, 3, 0,5 < a < 30, b = 1



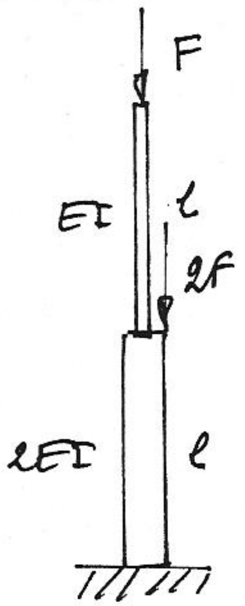
Invoer close-up grafiek:
1,5< a < 14



Opdracht 7

<u>Opdrachtformulering</u>

- Bepaal met behulp van de eindige elementenmethode en programma als Maple of Mathematica de bepalende kniklast.
- Bepaal met behulp van de potentiële energie de kniklast
- Geef een beschouwing over de nauwkeurigheid van de berekende resultaten.



<u>Uitwerking</u>

Kniklast met behulp van potentiële energie

Probeerfunctie: $w(0) = a_0 + a_1 x + a_2 x^2$

Bij $x = 0 \rightarrow w(0) = 0$ $\quad a_0 = 0$

Bij $x = 0 \rightarrow \frac{dw}{dx} = 0$ $\quad a_1 = 0$

Uiteindelijke probeerfunctie: $w(0) = a_2 x^2$

Inwendige potentiële energie:

$$E_{pot;i} = \frac{1}{2}\int_0^l 2EI\left(\frac{d^2w}{dx^2}\right)^2 dx + \frac{1}{2}\int_l^{2l} 2EI\left(\frac{d^2w}{dx^2}\right)^2 dx$$

$$E_{pot;i} = \frac{1}{2}\cdot 2EI\cdot(2a_2)^2\cdot l + \frac{1}{2}\cdot 2EI\cdot(2a_2)^2\cdot l = 6EI\cdot a_2{}^2\cdot l$$

Uitwendige potentiële energie:

$$E_{pot;u} = -\frac{1}{2}\int_0^l 3F\left(\frac{dw}{dx}\right)^2 dx - \frac{1}{2}\int_l^{2l} F\left(\frac{dw}{dx}\right)^2 dx$$

$$E_{pot;u} = -\frac{1}{2}\int_0^l 3F(2a_2 x)^2 dx - \frac{1}{2}\int_l^{2l} F(2a_2 x)^2 dx$$

$$E_{pot;u} = -F\cdot a_2{}^2 \cdot l^3\left(\frac{1}{2}\cdot 3\cdot 4\cdot\frac{1}{3}+\frac{1}{2}\cdot 1\cdot 4\cdot\frac{1}{3}(8-1)\right) = -\frac{20}{3}F\cdot a_2{}^2\cdot l^3$$

Totale potentiële energie:

$$E_{pot} = E_{pot;i} + E_{pot;u} = 6EI\cdot a_2{}^2\cdot l - \frac{20}{3}F\cdot a_2{}^2\cdot l^3$$

$$\delta E_{pot} = 0 \quad \rightarrow \quad \frac{\delta E_{pot}}{\delta a_2} = 0$$

$$\delta E_{pot} = 12EI\cdot a_2\cdot l - \frac{40}{3}F\cdot a_2{}^2\cdot l^3 = 0 \quad \rightarrow \quad a_2 \neq 0 \rightarrow \quad \boxed{F_k = \frac{9EI}{10l^2}}$$

Kniklast met behulp van eindig elementenmethode

Elementstijfheidsmatrix

$$\delta W_{I;1} = -[\delta v]^T EI\begin{bmatrix} \frac{12}{l^3} & \frac{6}{l^2} & -\frac{12}{l^3} & \frac{6}{l^2} \\ \frac{6}{l^2} & \frac{4}{l} & -\frac{6}{l^2} & \frac{2}{l} \\ -\frac{12}{l^3} & -\frac{6}{l^2} & \frac{12}{l^3} & -\frac{6}{l^2} \\ \frac{6}{l^2} & \frac{2}{l} & -\frac{6}{l^2} & \frac{4}{l} \end{bmatrix} v \qquad \delta W_{I;2} = -[\delta v]^T EI\begin{bmatrix} \frac{24}{l^3} & \frac{12}{l^2} & -\frac{24}{l^3} & \frac{12}{l^2} \\ \frac{12}{l^2} & \frac{8}{l} & -\frac{12}{l^2} & \frac{4}{l} \\ -\frac{24}{l^3} & -\frac{12}{l^2} & \frac{24}{l^3} & -\frac{12}{l^2} \\ \frac{12}{l^2} & \frac{4}{l} & -\frac{12}{l^2} & \frac{8}{l} \end{bmatrix} v$$

$$\delta W_{I;tot} = -[\delta v]^T EI\begin{bmatrix} \frac{12}{l^3} & \frac{6}{l^2} & -\frac{12}{l^3} & \frac{6}{l^2} & 0 & 0 \\ \frac{6}{l^2} & \frac{4}{l} & -\frac{6}{l^2} & \frac{2}{l} & 0 & 0 \\ -\frac{12}{l^3} & -\frac{6}{l^2} & \frac{36}{l^3} & \frac{6}{l^2} & -\frac{24}{l^3} & \frac{12}{l^2} \\ \frac{6}{l^2} & \frac{2}{l} & \frac{6}{l^2} & \frac{12}{l} & -\frac{12}{l^2} & \frac{4}{l} \\ 0 & 0 & -\frac{24}{l^3} & -\frac{12}{l^2} & \frac{24}{l^3} & -\frac{12}{l^2} \\ 0 & 0 & \frac{12}{l^2} & \frac{4}{l} & -\frac{12}{l^2} & \frac{8}{l} \end{bmatrix}\begin{bmatrix} w_1 \\ \phi_1 \\ w_2 \\ \phi_2 \\ w_3 \\ \phi_3 \end{bmatrix}$$

Geometriestijfheidsmatrix

$$\delta W_{E;1}=[\delta v]^T F\begin{bmatrix}\frac{6}{5l} & \frac{1}{10} & -\frac{6}{5l} & \frac{1}{10}\\ \frac{1}{10} & \frac{2l}{15} & -\frac{1}{10} & -\frac{l}{30}\\ -\frac{6}{5l} & -\frac{1}{10} & \frac{6}{5l} & -\frac{1}{10}\\ \frac{1}{10} & -\frac{l}{30} & -\frac{1}{10} & \frac{2l}{15}\end{bmatrix}v \qquad \delta W_{E;2}=[\delta v]^T F\begin{bmatrix}\frac{18}{5l} & \frac{3}{10} & -\frac{18}{5l} & \frac{3}{10}\\ \frac{3}{10} & \frac{6l}{15} & -\frac{3}{10} & -\frac{l}{10}\\ -\frac{18}{5l} & -\frac{3}{10} & \frac{18}{5l} & -\frac{3}{10}\\ \frac{3}{10} & -\frac{l}{10} & -\frac{3}{10} & \frac{6l}{15}\end{bmatrix}v$$

$$\delta W_{E;tot}=-[\delta v]^T F\begin{bmatrix}\frac{6}{5l} & \frac{1}{10} & -\frac{6}{5l} & \frac{1}{10} & 0 & 0\\ \frac{1}{10} & \frac{2l}{15} & -\frac{1}{10} & -\frac{l}{30} & 0 & 0\\ -\frac{6}{5l} & -\frac{1}{10} & \frac{24}{5l} & \frac{1}{5} & -\frac{18}{5l} & \frac{3}{10}\\ \frac{1}{10} & -\frac{l}{30} & \frac{1}{5} & \frac{8l}{15} & -\frac{3}{10} & -\frac{l}{10}\\ 0 & 0 & -\frac{18}{5l} & -\frac{3}{10} & \frac{18}{5l} & -\frac{3}{10}\\ 0 & 0 & \frac{3}{10} & -\frac{l}{10} & -\frac{3}{10} & \frac{6l}{15}\end{bmatrix}\begin{bmatrix}w_1\\ \phi_1\\ w_2\\ \phi_2\\ w_3\\ \phi_3\end{bmatrix}$$

$w_3=0$ $\phi_3=0$, wegstrepen matrix

$$DET\left(-EI\begin{bmatrix}\frac{12}{l^3} & \frac{6}{l^2} & -\frac{12}{l^3} & \frac{6}{l^2}\\ \frac{6}{l^2} & \frac{4}{l} & -\frac{6}{l^2} & \frac{2}{l}\\ -\frac{12}{l^3} & -\frac{6}{l^2} & \frac{36}{l^3} & \frac{6}{l^2}\\ \frac{6}{l^2} & \frac{2}{l} & \frac{6}{l^2} & \frac{12}{l}\end{bmatrix}+F\begin{bmatrix}\frac{6}{5l} & \frac{1}{10} & -\frac{6}{5l} & \frac{1}{10}\\ \frac{1}{10} & \frac{2l}{15} & -\frac{1}{10} & -\frac{l}{30}\\ -\frac{6}{5l} & -\frac{1}{10} & \frac{24}{5l} & \frac{1}{5}\\ \frac{1}{10} & -\frac{l}{30} & \frac{1}{5} & \frac{8l}{15}\end{bmatrix}\right)=0$$

$$DET\left(\begin{bmatrix}-\frac{12}{l^3}EI+\frac{6}{5l}F & -\frac{6}{l^2}EI+\frac{1}{10}F & \frac{12}{l^3}EI-\frac{6}{5l}F & -\frac{6}{l^2}EI+\frac{1}{10}F\\ -\frac{6}{l^2}EI+\frac{1}{10}F & -\frac{4}{l}EI+\frac{2l}{15}F & \frac{6}{l^2}EI-\frac{1}{10}F & -\frac{2}{l}EI-\frac{l}{30}F\\ \frac{12}{l^3}EI-\frac{6}{5l}F & \frac{6}{l^2}EI-\frac{1}{10}F & -\frac{36}{l^3}EI+\frac{24}{5l}F & -\frac{6}{l^2}EI+\frac{1}{5}F\\ -\frac{6}{l^2}EI+\frac{1}{10}F & -\frac{2}{l}EI-\frac{l}{30}F & -\frac{6}{l^2}EI+\frac{1}{5}F & -\frac{12}{l}EI+\frac{8l}{15}F\end{bmatrix}\right)=0$$

Delen van "Φ_1" en "Φ_2" door "l"

$$DET\left(\begin{bmatrix} -\frac{12}{l^3}EI+\frac{6}{5l}F & -\frac{6}{l^3}EI+\frac{1}{10l}F & \frac{12}{l^3}EI-\frac{6}{5l}F & -\frac{6}{l^3}EI+\frac{1}{10l}F \\ -\frac{6}{l^2}EI+\frac{1}{10}F & -\frac{4}{l^2}EI+\frac{2}{15}F & \frac{6}{l^2}EI-\frac{1}{10}F & -\frac{2}{l^2}EI-\frac{1}{30}F \\ \frac{12}{l^3}EI-\frac{6}{5l}F & \frac{6}{l^3}EI-\frac{1}{10l}F & -\frac{36}{l^3}EI+\frac{24}{5l}F & -\frac{6}{l^3}EI+\frac{1}{5l}F \\ -\frac{6}{l^2}EI+\frac{1}{10}F & -\frac{2}{l^2}EI-\frac{1}{30}F & -\frac{6}{l^2}EI+\frac{1}{5}F & -\frac{12}{l^2}EI+\frac{8}{15}F \end{bmatrix}\right)=0$$

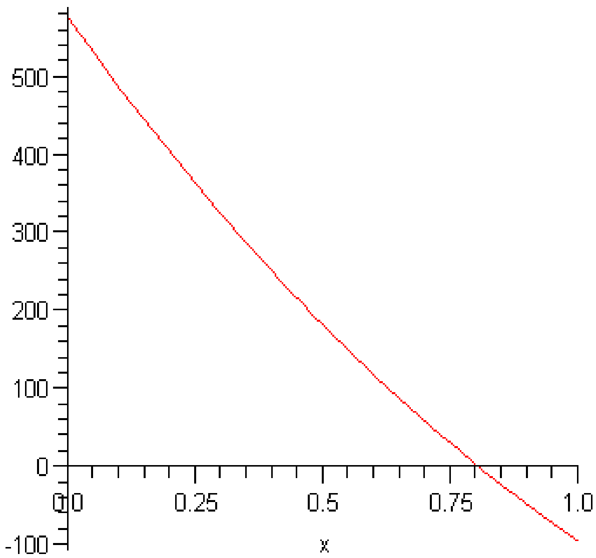
Bepalen van eigenwaarde: $\lambda=\frac{Fl^2}{EI}$, door wegdelen van "EI" en "l"

$$DET\left(\begin{bmatrix} -12+\frac{6\lambda}{5} & -6+\frac{\lambda}{10} & 12-\frac{6\lambda}{5} & -6+\frac{\lambda}{10} \\ -6+\frac{\lambda}{10} & -4+\frac{2\lambda}{15} & 6-\frac{\lambda}{10} & -2-\frac{\lambda}{30} \\ 12-\frac{6\lambda}{5} & 6-\frac{\lambda}{10} & -36+\frac{24\lambda}{5} & -6+\frac{\lambda}{5} \\ -6+\frac{\lambda}{10} & -2-\frac{\lambda}{30} & -6+\frac{\lambda}{5} & -12+\frac{8\lambda}{15} \end{bmatrix}\right)=0$$

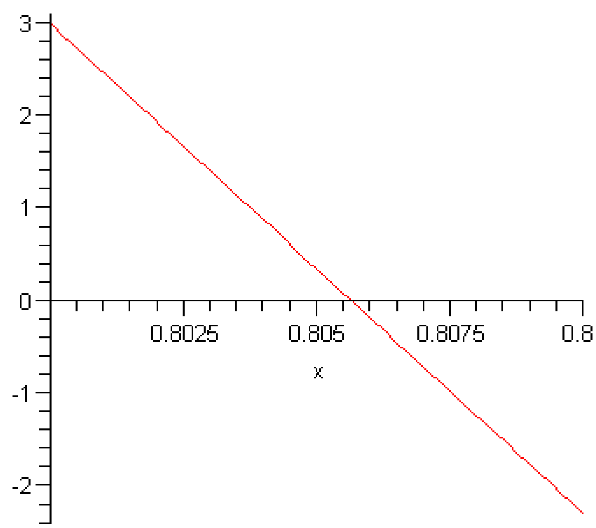
Uitwerking determinant

$$576-912x+\frac{6441}{25}x^2-\frac{821}{50}x^3+\frac{21}{80}x^4$$

Plotten van eigenwaarde (met x tussen 0 en 1)



Plotten van eigenwaarde (met x tussen 0,8 en 0,81)



De eigenwaarde heeft een waarde van 0,806

$$\boxed{F_k=0{,}806\frac{EI}{l^2}}$$

De waarde van de eindige elementenmethode is nauwkeuriger dan met behulp van de potentiële energie door gebruik te maken kwadratische oplosfuncties. De eindige elementenmethode komt dichter bij de realiteit.